NON-TRADED GOODS AND
REAL EXCHANGE RATE VOLATILITY[*]

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Abstract

I study the role of non-traded goods in explaining the high real exchange rate volatility observed in the data. Previous studies have found that most of the variance of the real exchange rate is caused by deviations from purchasing power parity for traded-goods prices, while changes in the relative price of non-traded goods play a negligible role. Using a structural VAR model I find that sector-specific productivity shocks in the traded and non-traded-goods sectors can explain a large fraction of the variation in real exchange rates. Then I develop a DSGE model with sticky prices and endogenous monetary policy that reconciles these two pieces of evidence. In the model sector-specific productivity shocks are able to generate real exchange rate volatilities as high as observed in the data without causing excessive volatility in the relative price of non-traded goods.

Keywords: Non-traded goods, Real exchange rate

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1 Introduction

Real exchange rates exhibit a large amount of short-term volatility while at the same time they are highly persistent. Rogoff (1996) refers to this phenomenon as the PPP puzzle. For example, Chari et al. (2002) find that the volatility of the real exchange rate is on average four to five times larger than the volatility of output. Bergin and Feenstra (2001) find similar results, with some countries exhibiting volatilities even higher[1]. Both studies estimate the autocorrelation coefficient at around 0.8. However it has been difficult to account for both the high volatility and high persistence in a unified framework. In DSGE models with sticky prices monetary shocks are capable of generating the observed volatility –through Dornbusch's overshooting effect– but cannot explain the persistence (Chari et al. (2002)). On the other hand, persistent productivity shocks can generate persistent real exchange rates, but cannot explain their volatility.

In this paper I study the role of non-traded goods in explaining the high volatility and high persistence of real exchange rates. Engel (1999) and others[2] have performed real exchange rate decomposition exercises and have found that changes in the relative price of non-traded goods account only for a negligible amount of the variability of real exchange rates. They find that almost all of the variation of the real exchange rate is caused by deviations from purchasing power parity for traded goods. This result has been used to argue that non-traded goods may not play a fundamental role for the dynamics of the real exchange rate. However the result depends on the exact methodology used. In Section 2 I describe how some refinements in the methodology can significantly increase the fraction of real exchange rate variance attributed to changes in the relative price of non-traded goods. More importantly, the previous conclusion is misguided. Non-traded goods may be important for the dynamics of the real exchange rate even if the relative price of non-traded goods exhibits little volatility. If productivity shocks are imperfectly correlated across traded and non-traded goods and price stickiness prevents the relative price of non-traded goods from adjusting, then these productivity shocks may be responsible for amplifying the response of the nominal and real exchange rates, causing large deviations from PPP for *traded-goods* prices. My empirical strategy consist of estimating a structural vector autoregressive (VAR) model in which sector-specific

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[1] In both cases the data is in logarithms and HP-filtered.

[2] See Mendoza (2005) for a survey.

productivity shocks are identified by using long-run restrictions based on the Balassa-Samuelson effect and the assumption that small economies cannot affect world variables. I find that a large fraction of the variance of the real exchange rate can be attributed to these innovations. This analysis is similar to Clarida and Gali (1994) empirical study of the importance of monetary shocks for real exchange rate fluctuations, although their identification scheme is very different since it is meant to identify another type of shocks.

My next step consists of studying the plausibility of sector-specific productivity shocks explaining the high volatility of the real exchange rate by simulating a dynamic stochastic general equilibrium (DSGE) model. Several studies have introduced models with non-traded goods to explain some empirical inconsistencies of one-good open economy models. For example Backus and Smith (1993) incorporate non-traded goods in a two-country model to study the small correlation of consumption across countries and the existence of large interest-rate differentials. Stockman and Tesar (1995) study the role of non-traded goods in explaining the behavior of consumption, investment, and the trade balance. Benigno and Thoenissen (2004) develop a model with non-traded goods and incomplete markets to explain the lack of correlation between the real exchange rate and relative consumption across countries. While these and other papers address the role of non-traded goods in explaining the international co-movements of certain aggregate variables, they do not attempt to explain the large volatility of the real exchange rate relative to output.

The evidence from real exchange rate decomposition exercises has favored the development of models in which real exchange rate fluctuations are driven by monetary shocks and deviations of purchasing power parity for traded goods. Typically sticky-price models in which real exchange rate fluctuations are driven by monetary shocks are able to match the high volatility of the real exchange rate observed in the data, but they cannot explain its large persistence (Chari et al. (2002)). Benigno (2004), however, shows that monetary shocks can generate persistent and volatile real exchange rates when monetary policy is endogenous, without assuming an unreasonable degree of price stickiness. However these monetary models require the assumption of a high degree of risk aversion.

In addition to increase the persistence of the real exchange rate after a monetary shock, the introduction of endogenous monetary policy can also magnify the volatility of the real exchange rate following a productivity shock. In this paper I develop a model with sticky prices and endogenous monetary policy in which sector-specific productivity shocks are able to generate high real exchange rate volatility. A similar model by Corsetti et al. (2005) also uses sector-specific productivity shocks to generate highly volatile exchange rates. However they assume either an unreasonably high degree of risk aversion or an unreasonably low elasticity of substitution between domestic and foreign traded goods. My model shows that the presence of non-traded goods per se actually reduces the volatility of the exchange rate. It is the low correlation between productivity shocks across traded and non-traded goods that can increase the volatility of the real exchange rate by reducing the correlation between domestic and foreign consumption.

The paper proceeds as follows. In Section 2 I review the role of the relative price of non-traded goods in real exchange rate decompositions. In Section 3 I estimate a structural VAR model and compute the percentage of the forecast-error variance attributable to sector-specific productivity shocks. In Section 4 I present the theoretical model. In Section 5 I calibrate and simulate the model and present the results. Section 6 concludes.

2 Real Exchange Rate Decompositions: How Important are Changes in the Relative Price of Non-Traded Goods?

In this section I briefly review the methodology used to account for the importance of changes in the relative price of non-traded goods in explaining movements of the real exchange rate and present some empirical results.

The real exchange rate (RER), defined as the price of the foreign consumption basket in terms of the domestic consumption basket can be expressed as

$$RER = \frac{SP^*}{P} \tag{1}$$

where P and P^* are the domestic and foreign consumption price levels and S is the nominal exchange rate. The RER can be decomposed as follows

$$RER = \frac{SP^{T*}}{P^T} \frac{P^* / P^{T*}}{P / P^T} \tag{2}$$

where the first term is the price of foreign traded goods in terms of domestic traded goods (RER^T), and the second term is the ratio of the foreign relative price of non-traded goods to the domestic relative price of non-traded goods (RER^N). Taking logarithms we obtain

$$rer = rer^T + rer^N \tag{3}$$

where $rer \equiv \log(RER)$. Then we can decompose the variance as

$$\mathrm{var}(rer) = \mathrm{var}(rer^T) + \mathrm{var}(rer^N) + 2\,\mathrm{cov}(rer^T, rer^N) \tag{4}$$

One can extract two important observations from these accounting exercises. First, the importance of the relative price of non-traded goods increases when the volatility of the nominal exchange rate is low. For example, Mendoza (2000, 2005) notices that there is a substantial difference in the variance decomposition for the US/Mexican real exchange rate during the period of managed exchange rate and during the floating period. This observation also holds for other economies. This phenomenon can be rationalized by assuming that the nominal exchange rate is very volatile and prices are sticky. Under complete price stickiness nominal exchange rate movements are translated one-for-one into the tradeable RER. This, however, cannot explain why at long horizons, when prices become flexible, Engel's results doesn't change.

The second important observation is that the results are sensitive to the measure of tradeable prices. Some authors have used the CPI for goods or the PPI. However, Burstein et al. (2005) suggest using import and export price indexes instead. Their argument is that CPI for goods includes the cost of non-traded distribution and retail services, while import and export price indexes measure prices at the dock. Moreover, they measure the prices of goods actually traded rather than assuming that all goods are tradeable. They find that this approach increases the contribution of the relative price of non-traded goods up to fifty percent of the total variance.

In addition to the choice of the proxies for traded-goods and overall price levels, authors have differed in the statistic used. Roger and Jenkins (1995) and Engel (1999) compute the mean square error instead of the variance. This is a better approach when the components have a significant trend. Authors have also diverged on how to assign the covariance term between the two components.

I perform a variance decomposition exercise similar to those made by other authors. The results are presented in Table 1. I use two different measures of traded-goods prices: the PPI and the geometrical average of the export and import price index (shown in Table 1 as "PT"). In addition I perform a decomposition using the effective (trade-weighted) rather than bilateral real exchange rate (shown in Table 1 as "REER"). In this case the measure of traded goods is the geometrical average of the EPI and IPI. The flaw of this third decomposition is that it does not take into account the price changes in the trade partners. The overall consumption price level is measured by the CPI. The fraction of the covariance term assigned to each component is proportional to its variance. Therefore, the fraction assigned to *h*-quarter changes of the non-traded component is

$$\frac{\mathrm{var}(\Delta_h rer_t^N)}{\mathrm{var}(\Delta_h rer_t^N)+\mathrm{var}(\Delta_h rer_t^T)} \tag{5}$$

As seen in Table 1 the choice of the measure of traded-goods prices is crucial. In all the countries studied and for all horizons using the geometric average of export and import prices instead of the PPI increases the fraction of real exchange rate variance assigned to the relative price of non-traded goods. In several countries the increase is dramatic. This shows how sensitive these decompositions are to the measure of traded-goods prices. For some countries the decomposition of the real effective exchange attributes more importance to the relative price of non-traded goods than the decomposition of the bilateral real exchange rate.

		h=1	**h=4**	**h=8**			**h=1**	**h=4**	**h=8**
Australia	**PPI**	7.8	9.7	15.1	**Italy**	**PPI**	3.9	3.5	3.7
	PT	69.7	69.3	65.0		**PT**	46.5	56.4	48.4
	REER	56.4	59.4	42.7		**REER**	33.5	30.1	29.5
Canada	**PPI**	36.4	49.9	53.4	**Norway**	**PPI**	27.5	16.8	19.8
	PT	71.5	78.9	77.2		**PT**	33.4	35.5	37.9
	REER	54.4	49.9	48.2		**REER**	40.1	41.1	43.0
Denmark	**PPI**	7.2	6.5	5.5	**Spain**	**PPI**	5.3	3.3	3.2
	PT	9.2	11.2	12.6		**PT**	17.5	13.8	17.0
	REER	52.4	69.0	67.5		**REER**	27.2	21.8	22.0
Finland	**PPI**	4.3	3.1	2.7	**Sweden**	**PPI**	9.0	8.3	8.8
	PT	8.5	10.2	9.6		**PT**	14.5	17.2	16.6
	REER	14.6	16.5	15.0		**REER**	38.4	42.4	40.7

Table 1: Percentage of RER Variance Attributed to Changes of the

Relative Price of Non-Traded Goods at Different Horizons

Nevertheless, the importance of non-traded goods for real exchange rate fluctuations may not be correctly assessed by looking at movements of the relative price of non-traded goods. The fact that some goods are not traded not only implies that some of the real exchange rate movements will be caused by changes in the relative price of non-traded goods, but it also may have a role in explaining fluctuations in the price of traded goods across countries.

3 Structural VAR Model: How Important are Sector-Specific Productivity Shocks?

The presence of non-traded goods implies that productivity shocks may affect the traded-goods sector and the non-traded-goods sectors differently. Regardless of whether one believes that economy-wide shocks may have a different impact on each sector or that there exist sector-specific shocks the result is the same: exogenous changes in relative productivity across sectors. The decomposition exercises discussed in the previous section ignore the cause of the changes in the relative price of non-traded goods and the deviations from PPP for traded goods. In this section I study how much of the variation of the real exchange rate can be attributed to innovations in the relative productivity of the two sectors, regardless of their impact on the relative price of non-traded goods. The methodology consists on estimating a structural VAR in which the identification scheme is meant to capture those innovations, and compute their relative contribution to the real exchange rate variance.

3.1 Data

I perform the empirical analysis using quarterly data from a group of small OECD economies. The variables used are contained in the vector Z_t

$$Z_t = \left[GDP_t, R_t, NX_t, Y_t^H / Y_t^N, RPN_t, TOT_t, REER_t \right]' \qquad (6)$$

where all variables are in logarithms. GDP_t is the gross domestic product. R_t is the gross real interest rate, measured by a short-term gross interest rate[3] divided by the gross expected inflation, which is calculated as the average of the inflation rates in the four preceding quarters. NX_t is the ratio of real exports to real imports.

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[3] The choice of the interest rate is based on data availability. For some countries I use the yield on a three-month Treasury bill, while in others I use a money market rate.

Y_t^H / Y_t^N is the ratio of tradeable GDP to non-tradeable GDP. I consider goods as tradeable, and construction and services as non-tradeable[4]. RPN_t is the relative price of non-traded goods, measured as the ratio of CPI to the geometric average of the import and export price indexes[5]. TOT_t is the terms of trade, measured as the ratio of import to export price indexes. $REER_t$ is the CPI-based real effective exchange rate constructed by the IMF[6].

3.2 Identification and Estimation

I first estimate the following reduced-form VAR (p)

$$\sum_{j=0}^{p} A(j)\Delta Z_{t-j} = \alpha + v_t \tag{7}$$

where $A(0) = I$ and $v \sim N(0,\Omega)$. One can invert (7) to obtain its moving-average representation

$$\Delta Z = \chi + \sum_{j=0}^{\infty} B(j)v_{t-j} \tag{8}$$

where $\chi = \sum_{j=0}^{\infty} B(j)\alpha$. The model can be re-written in terms of its structural innovations,

$$\Delta Z_t = \chi + \sum_{j=0}^{\infty} C(j)e_{t-j} \tag{9}$$

where $e \sim N(0,I)$. Since $A(0) = I$ then $B(0) = I$ and $v_t = C(0)e_t$. Therefore identification of $C(0)$ allows us to recover the structural shocks from the residuals of the estimated VAR. In order to identify $C(0)$ notice that $Var(v) = C(0)Var(e)C(0)'$, which implies

$$\Omega = C(0)C(0)' \tag{10}$$

Since $C(j) = B(j)C(0)$ it follows that

$$\sum_{j=0}^{\infty} C(j) = \sum_{j=0}^{\infty} B(j)C(0) \tag{11}$$

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[4] I use sectorial output data from the OECD Quarterly National Accounts, which divides output into four sectors: Agriculture, hunting, forestry, and fishing; Mining, manufacturing, electricity, gas, and water; Construction; and Services.

[5] When import and export prices are not available I use unit export and import unit values instead.

[6] However I adjust it since they use the opposite convention to define the real exchange rate.

I impose restrictions on the matrix of long-run multipliers, the left hand side of (11)[7]. Once I obtain $\hat{C}(0)$ I can construct estimates of e_t as $\hat{e}_t = \hat{C}(0)^{-1}\hat{v}_t$. The restrictions in (10) and (12) uniquely identify $C(0)$.

The restrictions on the matrix of long-run multipliers to identify the structural innovations are based on economic theory. My first assumption is based on one of the explanations of the Balassa-Samuelson effect: Under capital mobility across countries and labor mobility across sectors, in the long-run the relative price of non-traded goods depends only on the productivity differential across sectors. A derivation of this well-known result can be found in Mark (2001) and Obstfeld and Rogoff (1996). However I do not assume that domestic and foreign goods are perfect substitutes, which implies that terms of trade shocks may also have permanent effects on the relative price of non-traded goods. The previous assumption allows me to identify relative supply shocks as the innovations in the relative price and relative demand shocks as innovations in the ratio of tradeable and non-tradeable GDP. The second assumption is that for small open economies the real interest rate and the terms of trade are exogenous in the long run. The last restrictions are derived from the previous ones and the decomposition of the real exchange rate. The real exchange rate can be decomposed into its tradeable component, the domestic relative price of non-traded goods, and the foreign relative price of non-traded goods. Since the terms of trade and the relative price of non-traded goods are not affected by other variables' shocks in the long run, only these two variables' innovations and its own can have a permanent effect on the level of the real exchange rate. Innovations to the real exchange rate must be interpreted as changes in the foreign relative price of non-tradeables and deviations from PPP for tradeables not caused by TOT shocks. These assumptions imply the following restrictions on the matrix of long-run multipliers:

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[7] The long-run multipliers measure the cumulated effect of shocks on the levels of the variables. Restricting their value to zero means that the shock has no permanent effects on the level of the (non-stationary) variable. This identification method was first used in Blanchard and Quah (1989).

$$\sum_{j=0}^{\infty} C(j) = \begin{bmatrix} c_{11} & c_{12} & c_{13} & c_{14} & c_{15} & c_{16} & c_{17} \\ 0 & c_{22} & 0 & 0 & 0 & 0 & 0 \\ c_{31} & c_{32} & c_{33} & c_{34} & c_{35} & c_{36} & c_{37} \\ c_{41} & c_{42} & c_{43} & c_{44} & c_{45} & c_{46} & c_{47} \\ 0 & 0 & 0 & 0 & c_{55} & c_{56} & 0 \\ 0 & 0 & 0 & 0 & 0 & c_{66} & 0 \\ 0 & 0 & 0 & 0 & c_{75} & c_{76} & c_{77} \end{bmatrix} \tag{12}$$

The reduced-form VAR in equation (7) is estimated by ordinary least squares. I use two lags, as suggested by the Akaike, likelihood ratio, and FPE criteria. Then the estimated coefficients $\hat{A}(j)$ and the residuals $\hat{v}_t$ are used to estimate $C(0)$ and H using the identifying restrictions (10) and (12).

3.3 Forecast-Error Variance Decomposition

The variance decomposition (VD) exercises in Section 2 measure the percentage of variance attributable to movements in each of the components of a variable, without taking into account the underlying driving forces of these movements. By contrast, using a VAR I can compute the percentage of variance attributable to each of the forecast-errors, or innovations, in the structural VAR. My identification scheme allows to interpret these innovations. In the presence of price stickiness, relative supply shocks may take some time to be fully incorporated into relative price of non-traded goods. Before that adjustment may take place, the adjustment may take place through the nominal exchange rate. In that case a VD would attribute the real exchange rate movement to movements on the relative price between domestic and foreign traded goods, even though the underlying factor is a relative supply shock and the future adjustment of the relative price of non-traded goods. A FEVD would correctly attribute the RER movement to a relative supply shock.

The *h*-step ahead forecast error of ΔZ_t is

$$FE_h = \Delta Z_{t+h} - E_t(\Delta Z_{t+h}) = \sum_{j=0}^{h-1} C(j) e_{t+h-j} \tag{13}$$

Given the independence of the innovations the *h*-step ahead forecast-error variance of ΔZ_t is

$$\mathrm{var}(FE_h) = \sum\nolimits_0^{h-1} C(j)C(j)' \tag{14}$$

I can obtain the variance due to a particular innovation *k* as

$$\mathrm{var}(FE_{k,h}) = \sum\nolimits_0^{h-1} C(j)I_k C(j)' \tag{15}$$

where I_k is a matrix with 1 in its (k,k) cell and zeros elsewhere. Taking the limit of these expressions I can compute the unconditional variance decomposition. The results are presented in Table 2.

The results suggest that most of the variation of the real exchange rate in these economies is caused by innovations in the relative price of non-traded goods or in the terms of trade. For most countries a large fraction of the real exchange rate variance is left unexplained. It should be noticed that the identification strategy is meant to properly identify innovations in the relative productivity between traded and non-traded goods, and may not be appropriate to account for the importance of other important sources of fluctuations, like monetary shocks.

	GDP	***R***	***NX***	***YT/YN***	***RPN***	***TOT***	***REER***
Australia	1.1	0.0	1.3	10.8	53.3	23.9	9.5
Canada	0.0	2.5	0.0	0.4	51.4	12.1	33.5
Denmark	0.7	3.7	1.2	0.6	59.2	2.8	31.9
Finland	5.6	0.3	0.1	0.3	36.5	1.1	56.0
Italy	0.1	4.3	0.4	12.9	24.0	29.5	28.8
Norway	0.5	1.0	0.3	0.7	31.5	3.5	62.5
Spain	1.5	0.1	3.5	0.6	35.3	28.7	30.2
Sweden	0.5	1.8	0.6	8.9	10.3	74.5	3.4

Table 2: Unconditional FEVD for the Real Effective Exchange Rate (%)

4 DSGE Model

I develop a two-country model and study the effects of sector-specific productivity shocks on real exchange rate volatility. In the model each country produces two goods. Home country produces goods H (traded) and N (non-traded), while the Foreign country produces F (traded) and M (non-traded). Deviations from PPP for traded goods occur because prices are sticky in local currency. This implies that the law of one price may not hold for the same good sold at Home and abroad, and therefore PPP for traded goods may not hold even if there is no preference bias towards domestic goods.

Prices with an asterisk are denominated in Foreign currency. Goods with an asterisk are consumed by the Foreign country. For example, C^F denotes consumption of Foreign good F by Home consumers, and P^{H*} denotes the foreign-currency price at which Home good H sold in the Foreign market.

4.1 Households

In each country every household owns and works for one firm that produces a variety of a traded or non-traded good. Each variety is indexed by z. A fraction γ of the households produces goods that cannot be traded internationally (denoted by N or M), while the rest produces goods that can be traded (denoted by H or F). Goods are produced using labor (L). The production functions are

$$Y^j(z) = A^j L^j(z) \tag{16}$$

where $j = N, H, M, F$, and A^j is the marginal product of labor in each sector.

Each Home household's preferences are described by the following utility function

$$U_t^j(z) = E_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} \left[\frac{C_{t+\tau}^{\ 1-\sigma}}{1-\sigma} - \frac{L_{t+\tau}^j(z)^{1+\omega}}{1+\omega} \right] \tag{17}$$

for $j = H, N$, where β is the subjective discount factor and C is the consumption basket, which is identical across Home households[8]. The utility function for Foreign households is identical, except that C_t must be replaced by C_t^* and $j = F, M$. Preferences over the varieties of the same good are described by the following CES aggregator

$$C^j = \left[\int_0^1 C^j(z)^{(\varepsilon-1)/\varepsilon} dz \right]^{\varepsilon/(\varepsilon-1)} \tag{18}$$

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[8] Since all Home households exhibit the same preferences and face the same prices, they all choose the same basket of goods. In addition, complete markets will also imply that they consume the same quantities.

for $j = H, H^*, F, F^*, N, M^*$, where ε is the elasticity of substitution between varieties. Preferences between baskets of Foreign and Home traded goods are described by

$$C^T = \left[\kappa (C^H)^{(\varsigma-1)/\varsigma} + (1-\kappa)(C^F)^{(\varsigma-1)/\varsigma} \right]^{\varsigma/(\varsigma-1)} \tag{19}$$

$$C^{T*} = \left[\kappa (C^{F*})^{(\varsigma-1)/\varsigma} + (1-\kappa)(C^{H*})^{(\varsigma-1)/\varsigma} \right]^{\varsigma/(\varsigma-1)} \tag{20}$$

where ς denotes the elasticity of substitution between Foreign and Home traded goods and κ measures the degree of preference bias towards domestic goods. Similarly, preferences between baskets of non-traded and traded goods are described by

$$C = \left[\gamma (C^N)^{(\eta-1)/\eta} + (1-\gamma)(C^T)^{(\eta-1)/\eta} \right]^{\eta/(\eta-1)} \tag{21}$$

$$C^* = \left[\gamma (C^{M*})^{(\eta-1)/\eta} + (1-\gamma)(C^{T*})^{(\eta-1)/\eta} \right]^{\eta/(\eta-1)} \tag{22}$$

Price indexes for these baskets of goods can be derived by calculating their unit cost:

$$P^j = \left[\int_0^1 P^j(z)^{1-\varepsilon} dz \right]^{1/(1-\varepsilon)} \tag{23}$$

$$P^T = \left[\kappa (P^H)^{1-\varsigma} + (1-\kappa)(P^F)^{1-\varsigma} \right]^{1/(1-\varsigma)} \tag{24}$$

$$P = \left[\gamma (P^N)^{1-\eta} + (1-\gamma)(P^T)^{1-\eta} \right]^{1/(1-\eta)} \tag{25}$$

$$P^{T*} = \left[\kappa (P^{F*})^{1-\varsigma} + (1-\kappa)(P^{H*})^{1-\varsigma} \right]^{1/(1-\varsigma)} \tag{26}$$

$$P^* = \left[\gamma (P^{M*})^{1-\eta} + (1-\gamma)(P^{T*})^{1-\eta} \right]^{1/(1-\eta)} \tag{27}$$

where $j = H, H^*, F, F^*, N, M^*$. These preferences imply the following demand functions for each variety:

$$C^H(z) = \kappa(1-\gamma) \left(\frac{P^H(z)}{P^H} \right)^{-\varepsilon} \left(\frac{P^H}{P^T} \right)^{-\varsigma} \left(\frac{P^T}{P} \right)^{-\eta} C \tag{28}$$

$$C^F(z) = (1-\kappa)(1-\gamma) \left(\frac{P^F(z)}{P^F} \right)^{-\varepsilon} \left(\frac{P^F}{P^T} \right)^{-\varsigma} \left(\frac{P^T}{P} \right)^{-\eta} C \tag{29}$$

$$C^N(z) = \gamma\left(\frac{P^N(z)}{P^N}\right)^{-\varepsilon}\left(\frac{P^N}{P}\right)^{-\eta} C \tag{30}$$

$$C^{F*}(z) = \kappa(1-\gamma)\left(\frac{P^{F*}(z)}{P^{F*}}\right)^{-\varepsilon}\left(\frac{P^{F*}}{P^{T*}}\right)^{-\varsigma}\left(\frac{P^{T*}}{P^*}\right)^{-\eta} C^* \tag{31}$$

$$C^{H*}(z) = (1-\kappa)(1-\gamma)\left(\frac{P^{H*}(z)}{P^{H*}}\right)^{-\varepsilon}\left(\frac{P^{H*}}{P^{T*}}\right)^{-\varsigma}\left(\frac{P^{T*}}{P^*}\right)^{-\eta} C^* \tag{32}$$

$$C^{M*}(z) = \gamma\left(\frac{P^{M*}(z)}{P^{M*}}\right)^{-\varepsilon}\left(\frac{P^{M*}}{P^*}\right)^{-\eta} C^* \tag{33}$$

Households have access to a complete set of contingent securities. Let $Q_{t,t+1}$ denote the stochastic discount factor, and D_{t+1} denote the nominal payoff in period *t+1* of the portfolio held at the end of period *t*. Then the intratemporal budget constraint for each Home household is

$$P_t C_t + E_t\left\{Q_{t,t+1} D_{t+1}\right\} = D_t + W_t^j(z) L_t^j(z) + PR_t^j(z) \tag{34}$$

for $j = H, N$, where W and PR denote wage and profits, respectively. The first order conditions with respect to consumption and assets imply

$$\beta\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma}\left(\frac{P_t}{P_{t+1}}\right) = Q_{t,t+1} \tag{35}$$

Taking expectations on both sides we get a conventional stochastic Euler equation:

$$\beta(1+R_t)E_t\left\{\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma}\left(\frac{P_t}{P_{t+1}}\right)\right\} = 1 \tag{36}$$

where

$$(1+R_t) = \frac{1}{E_t\left\{Q_{t,t+1}\right\}} \tag{37}$$

is the gross return of a risk-less one-period nominal bond. Similarly, the equations for the Foreign consumers are

$$\beta\left(\frac{C_{t+1}^*}{C_t^*}\right)^{-\sigma}\left(\frac{P_t^*}{P_{t+1}^*}\right)\left(\frac{S_t}{S_{t+1}}\right)=Q_{t,t+1} \tag{38}$$

$$\beta(1+R_t^*)E_t\left\{\left(\frac{C_{t+1}^*}{C_t^*}\right)^{-\sigma}\left(\frac{P_t^*}{P_{t+1}^*}\right)\right\}=1 \tag{39}$$

where S is the nominal exchange rate. Combining (35) and (38) we obtain the typical risk-sharing condition

$$C_t=\xi C_t^*\left(\frac{S_tP_t^*}{P_t}\right)^{1/\sigma} \tag{40}$$

where ξ depends upon initial conditions. In addition we can combine the Home and Foreign Euler equations to obtain the interest parity condition

$$(1+R_t)E_t\left\{\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma}\left(\frac{P_t}{P_{t+1}}\right)\right\}=(1+R_t^*)E_t\left\{\left(\frac{C_{t+1}^*}{C_t^*}\right)^{-\sigma}\left(\frac{P_t^*S_t}{P_{t+1}^*S_{t+1}}\right)\right\} \tag{41}$$

Finally, the first order condition with respect to labor supply for a Home household is

$$\frac{W_t^j(z)}{P_t}C_t^{-\sigma}=L_t^j(z)^{\omega} \tag{42}$$

for $j=H,N$. A similar labor supply function holds for the Foreign households with C_t^* instead of C_t and $j=F,M$.

4.2 Price Setting

Prices are sticky as in Calvo (1983). Each period the probability of a firm being able to adjust its price is $1-\theta^j$, $j=H,H^*,F,F^*,N,M^*$. Therefore Home firms in the non-traded sector that are able to adjust its price will choose $\overline{P}_t^N(z)$ to maximize

$$E_t\sum_{\tau=0}^{\infty}(\beta\theta^N)^{\tau}\left[\left(\frac{C_{t+\tau}}{C_t}\right)^{-\sigma}\left(\frac{\overline{P}_t^N(z)}{P_{t+\tau}^N}-\frac{W_{t+\tau}^N(z)}{A_{t+\tau}^NP_{t+\tau}^N}\right)C_{t+\tau}^N(z)\right] \tag{43}$$

where

$$C_{t+\tau}^{N}(z)=\gamma\left(\frac{\overline{P}_{t}^{N}(z)}{P_{t+\tau}^{N}}\right)^{-\varepsilon}\left(\frac{P_{t+\tau}^{N}}{P_{t+\tau}}\right)^{-\eta}C_{t+\tau} \tag{44}$$

Each firm producing a traded good chooses two prices, one for the Home market and one for the Foreign market. Each price is set in the local currency of that market. Home firms will choose $\overline{P}_{t}^{H}(z)$ and $\overline{P}_{t}^{H*}(z)$ to maximize

$$E_{t}\sum_{\tau=0}^{\infty}(\beta\theta^{H})^{\tau}\left[\left(\frac{C_{t+\tau}}{C_{t}}\right)^{-\sigma}\left(\frac{\overline{P}_{t}^{H}(z)}{P_{t+\tau}^{H}}-\frac{W_{t+\tau}^{H}(z)}{A_{t+\tau}^{H}P_{t+\tau}^{H}}\right)C_{t+\tau}^{H}(z)\right] \tag{45}$$

and

$$E_{t}\sum_{\tau=0}^{\infty}(\beta\theta^{H*})^{\tau}\left[\left(\frac{C_{t+\tau}}{C_{t}}\right)^{-\sigma}\left(\frac{\overline{P}_{t}^{H*}(z)S_{t+\tau}}{P_{t+\tau}^{H}}-\frac{W_{t+\tau}^{H}(z)}{A_{t+\tau}^{H}P_{t+\tau}^{H}}\right)C_{t+\tau}^{H*}(z)\right] \tag{46}$$

Since all firms in the same sector face a similar problem it must be that $\overline{P}_{t}^{j}(z)=\overline{P}_{t}^{j}\ \forall z$. Foreign firms choose $\overline{P}_{t}^{F*}(z),\overline{P}_{t}^{F}(z),\overline{P}_{t}^{M*}(z)$ facing a similar pricing problem. The presence of many firms implies that every period only a fraction $(1-\theta^{j})$ of firms will be able to adjust their prices. This implies that the price indexes in each sector will evolve as follows

$$P_{t}^{j}=\left[\theta^{j}(P_{t-1}^{j})^{1-\varepsilon}+(1-\theta^{j})(\overline{P}_{t}^{j})^{1-\varepsilon}\right]^{1/(1-\varepsilon)} \tag{47}$$

4.3 International Relative Prices

The real exchange rate is defined as $RER=SP^{*}/P$. Two important relative prices are the relative price between traded and non-traded goods ($RPN=P^{N}/P^{T}$, $RPN^{*}=P^{M*}/P^{T*}$) and the relative price between Home and Foreign traded goods ($RPT=P^{F}/P^{H}$, $RPT^{*}=P^{H*}/P^{F*}$). Notice that this relative price is different from the terms of trade, since under local currency pricing the law of one price needs not to hold (

$P^F \neq P^{F*}S$). Deviations from the law of one price for Home traded goods can be measured by $RER^H = P^{H*}S/P^H$ and $RER^F = P^{F*}S/P^F$.

4.4 Monetary Policy

The central bank chooses its policy instrument, a short-term interest rate, following a rule that may take the general form

$$R_t = \delta\tilde{R} + (1-\delta)\left[\delta_i R_{t-1} + \delta_\pi E_t(\Pi_{t+1} - \tilde{\Pi}) + \delta_y Y_t + \delta_{rer} RER_t\right] \qquad (48)$$

where $\tilde{R}$ is the steady-state nominal interest rate, equal to the natural real interest rate plus the target inflation rate $\tilde{\Pi}$.

The coefficient on the previous interest rate δ_i reflects the fact that central banks prefer to smooth their interest rate adjustments. It is a key ingredient to generate real exchange rate persistence following a non-serially-correlated monetary shock (Benigno (2004)).

In my model the central bank must target a measure of real activity ($\delta_y > 0$) in order to generate high real exchange rate volatility following a productivity shock. In fact, under a strict inflation-targeting regime ($\delta_\pi > 0, \delta_i = \delta_y = \delta_{rer} = 0$) productivity shocks in the traded-goods sector have no effect on the real exchange rate if there is no home bias ($\kappa = 0.5$) and the degrees of price stickiness are similar across countries ($\theta^H = \theta^{H*}, \theta^F = \theta^{F*}$)[9]. The reason is the following. If $\theta^H = \theta^{H*}$ a shock to productivity in the domestic traded-good sector (A^H) will affect inflation of domestically-produced traded goods in the home and foreign markets (π_t^H and π_t^{H*}) in a similar way. Without home bias these two inflation rates have the same weight in the domestic and foreign overall inflation rates. Under a strict inflation-targeting regime the interest rates in both countries will change by the same amount, despite output will be higher in the Home country. Since there is no change in the interest-rate differential the nominal exchange rate remains constant. It follows that since the

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[9] This result is shown by Benigno (2004) in a model in which all goods are traded and there is no home bias. In my model it is clear that this result cannot be generalized when these two assumptions are dropped.

nominal exchange rate doesn't change and the inflation differential is zero, the real exchange rate must remain constant. In the case of home bias the weight of π_t^H and π_t^{H*} is different in each country, and that causes an inflation-rate differential. Similarly, a productivity shock in the domestic non-traded goods sector also creates an inflation differential.

5 Calibration and Simulation

5.1 Calibration

In order to calibrate and simulate the model I first log-linearize it around its steady state. The equations of the log-linearized model are presented in the full version of the paper, available in the author's website.

The baseline parameter values used to calibrate the model are presented in Table 3. They are standard in the international business cycle literature (e.g. Gali and Monacelli (2005), Mendoza (1991)). The parameters that depend on frequency are chosen to match quarterly frequency. The subjective discount factor is set at 0.99. The coefficient of constant relative risk aversion takes the value 2, which is consistent with empirical measures of relative risk aversion (Some authors argue that a value of at least 5 is necessary to match the volatility of the real exchange rate[10]). The parameter ω is set at 3, which implies an elasticity of labor supply equal to 1/3. The elasticity of substitution across traded and non-traded goods is 0.5, while between domestic and foreign traded goods is 1.5. The elasticity of substitution across varieties is 6, which implies a steady-state mark-up of 20 percent. The probability of not adjusting the price is 0.75, which implies that firms on average change their price every four quarters. I assume that the central banks follow a standard Taylor-rule without interest-rate smoothing or real exchange rate targeting ($\delta_i = \delta_{rer} = 0$). The coefficient on inflation is set at 1.5, consistent with the Taylor principle of responding to inflation more than one for one, and the coefficient on output is set at 0.5.

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[10] Chari et al. (2002) set σ =5, and in Benigno (2004) σ =10.

σ	Coefficient of Relative Risk Aversion	2
β	Subjective Discount Factor	0.99
ω	Inverse of the Elasticity of Labor Supply	3
θ	Probability of no price adjustment	0.75
η	EoS between Traded and Non-Traded Goods	0.5
γ	Bias towards Non-Traded Goods	0.5
ζ	EoS between Home and Foreign Traded Goods	1.5
κ	Home Bias in Traded goods	0.5
ε	EoS between Varieties	6
δ_π	Coefficient on Inflation	1.5
δ_y	Coefficient on Output	0.5

Table 3: Calibration Parameters

5.2 Simulation Exercises

The first exercise consists of simulating the model under the baseline parameters for different degrees of correlation between productivity shocks across sectors. For each correlation I compute the volatility of the real exchange rate, measured as its standard deviation divided by the standard deviation of output. Figure 1 shows the results for two different values of the output coefficient on the Taylor rule. For the baseline output coefficient the volatility of the real exchange rate relative to output is almost twice as large when productivity shocks across sectors are completely uncorrelated as when they are perfectly correlated. Sector-specific productivity shocks help reduce the correlation between domestic and foreign consumption. This in turn increases the volatility of the real exchange rate. Notice that the presence of non-traded goods alone cannot have the same effect. In fact, as shown later in Figure 2 the presence of non-traded goods actually reduces the volatility of the real exchange rate.

It is evident from Figure 1 that the coefficient on output in the Taylor rule has a very important effect on the volatility of the real exchange rate, even if shocks are perfectly correlated across sectors. As the central bank reduces the volatility of output it increases the volatility of the interest rate, which in turn increases the volatility of the real exchange rate through the interest parity condition.

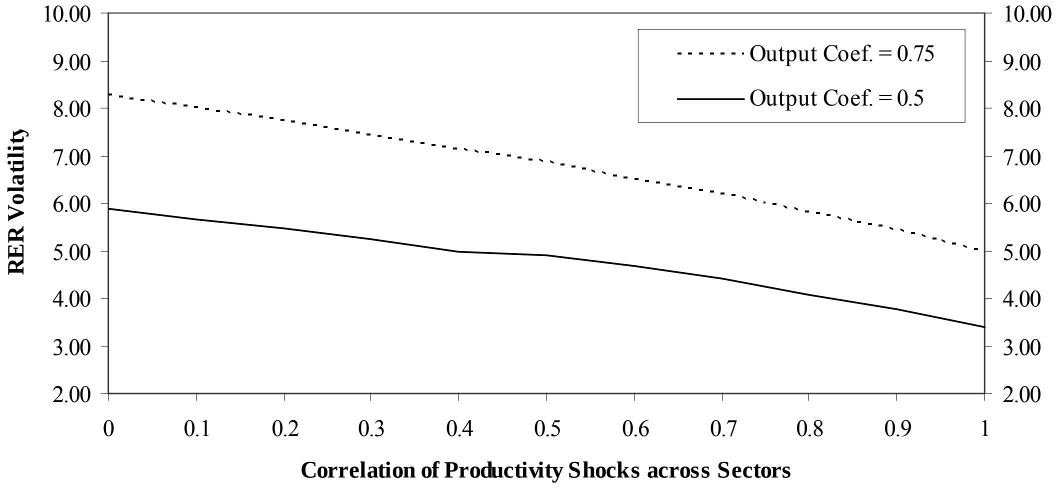

Figure 1: RER Volatility and the Correlation of Productivity Shocks

The following simulation exercises consist of changing some key parameters under two different assumptions: perfect correlation and no correlation of productivity shocks across sectors. Figure 2 shows the effect that changing the weight of non-traded goods in the consumption basket has on real exchange rate volatility. When productivity shocks across sectors are perfectly correlated increasing the weight of non-traded goods reduces the volatility of the real exchange rate. This simply indicates that other things equal less open economies will have less volatile exchange rates. On the other hand, when shocks across sectors are uncorrelated there is an additional and opposing effect. As the weight of non-traded goods increases the correlation between domestic and foreign overall consumption falls, increasing the volatility of the real exchange rate. These two effects cause the inverted-U shape shown in Figure 2. The highest volatility occurs when $\gamma = 0.6$, so that the traded and non-traded sectors have fairly similar weights.

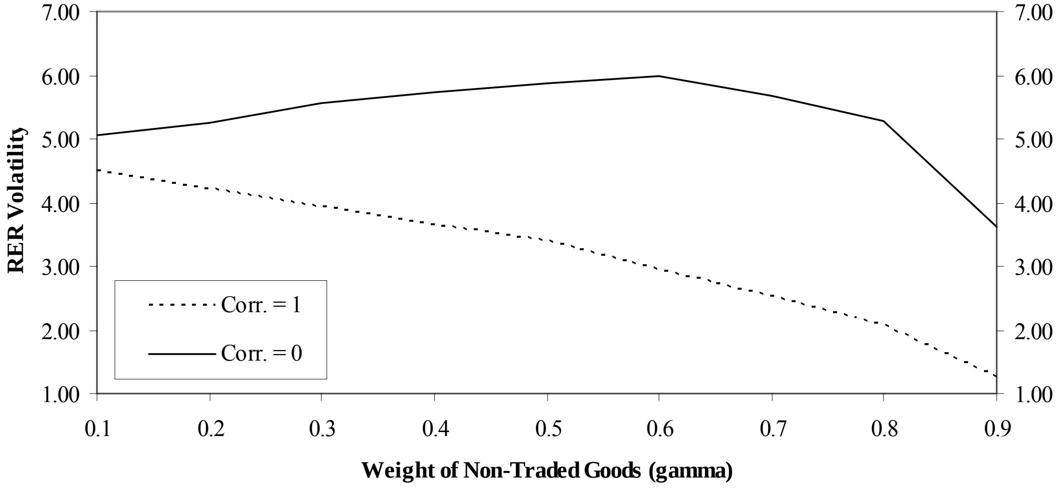

Figure 2: RER Volatility and the Weight of Non-Traded Goods

Another determinant of the degree of openness of the economies is their preference for domestically produced traded goods ("home bias"). Figure 3 shows that, regardless of the correlation of productivity shocks across sectors, more open economies exhibit more volatile exchange rates. The real exchange rate is more volatile when productivity shocks are imperfectly correlated for any degree of home bias.

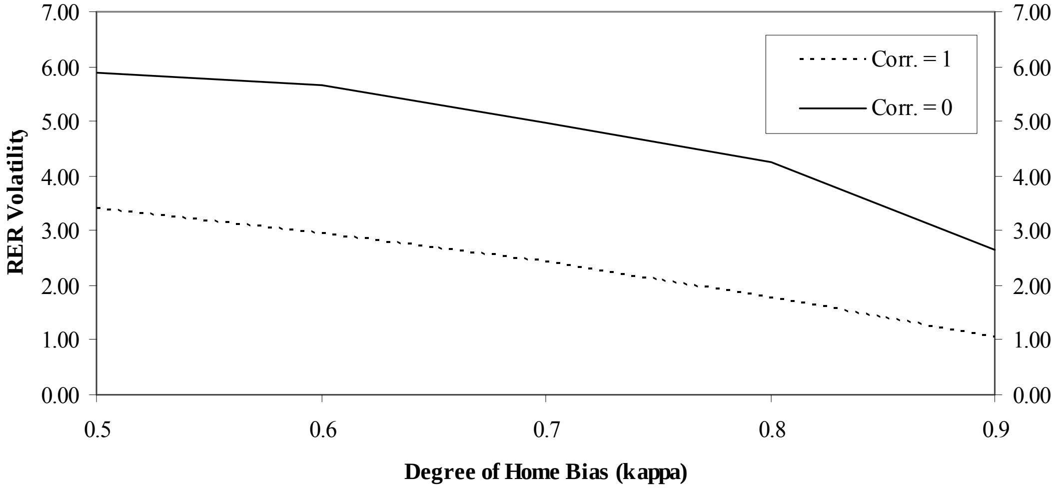

Figure 3: RER Volatility and the Degree of Home Bias in Traded Goods

The next parameter to be studied is the degree of price stickiness, measured by the probability that a firm cannot adjust its price in a given period (θ). A higher degree of price stickiness reduces the magnitude of the response of output and the real exchange rate to productivity shocks, but the overall effect is an increase of the volatility of the real exchange rate relative to that of output (see Figure 4).

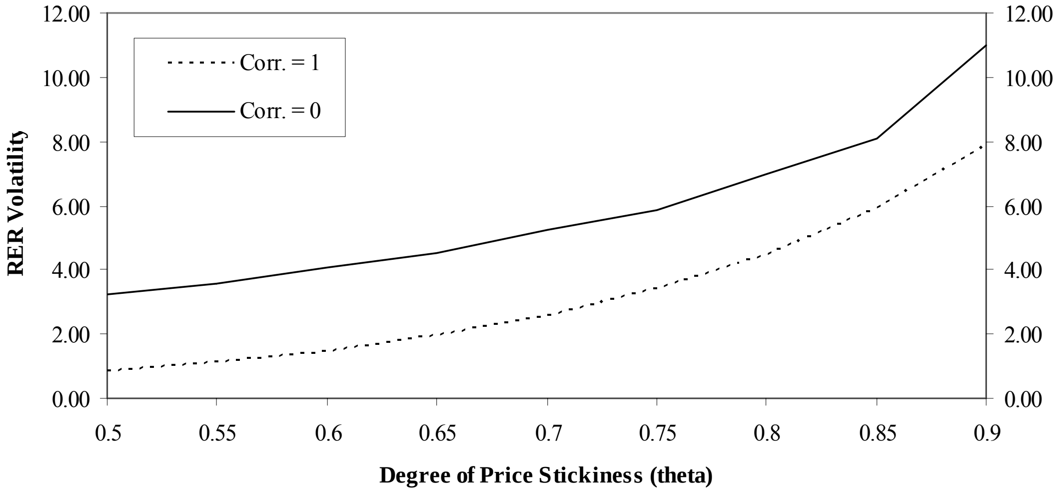

Figure 4: RER Volatility and the Degree of Price Stickiness

The model allows specifying different degrees of price stickiness across countries and across sectors. Figure 5 shows how the volatility of the exchange rate changes when the difference between $\theta^{H}, \theta^{H*}, \theta^{F}, \theta^{F*}$ and θ^{N}, θ^{M} increases, while keeping the average θ equal to 0.75. The relationship is non-monotonic, but for large changes it seems to be positive, as prices of traded goods become relatively stickier the real exchange rate becomes more volatile. Recall that it is precisely price stickiness in the traded sector the cause of deviations of the law of one price for similar goods across countries.

A variance decomposition exercise (as in Section 2) of simulated time series indicates that only 33% of the variance of the real exchange rate can be attributed to changes in the relative price of non-traded goods. This number is similar to the empirical results obtained when using actually-traded goods prices. Moreover, since in this model non-traded goods play a crucial role, it points out that the importance of non-traded goods for real exchange rate fluctuations may be beyond just changes in the relative price of non-traded goods.

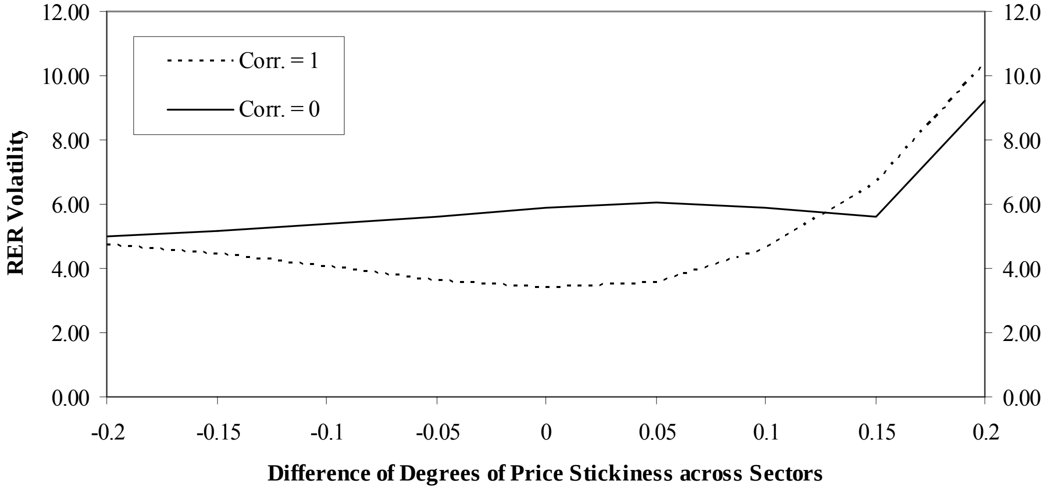

Figure 5: RER Volatility and Relative Price Stickiness across Sectors

6 Concluding Remarks

The empirical evidence suggests that most of the variation of the real exchange rate can attributed to deviations from purchasing power parity for traded goods rather than to changes in the relative prices of non-traded goods. This has led some economists to argue that the presence of non-traded goods may be of little importance to explain real exchange rate volatility. By using a structural VAR model I showed that a large fraction of the real exchange rate can be attributed to productivity shocks that cause changes in the relative productivity of non-traded goods.

The simulation exercises of my theoretical model reconcile these two pieces of evidence. They show that changes in relative productivity across traded and non-traded goods can explain the high volatility of the real exchange rate. This implies that real shocks can explain both the high volatility and high persistence of real exchange rate fluctuations, despite the common view that monetary shocks are necessary to explain the high volatility of the real exchange rate. Moreover most of this volatility is caused by deviations from PPP for traded-goods prices, not by changes in the relative price of non-traded goods, which is consistent with the evidence from real exchange rate decomposition exercises.

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